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Linear Equation and Linear Function: An Alternative Material Based on Visual Representation in Mathematics

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Abstrak

Persamaan linear dan fungsi linear merupakan salah materi yang penting bagi siswa dalam mengembangkan pengetahuan aritmetika dan aljabar mereka secara mendalam. Namun faktanya terdapat gap antara *potential development* dan *actual development* yang menyebabkan hambatan bagi siswa dalam memahami konsep kedua materi tersebut. Gap tersebut berakibat pada hambatan-hambatan lainnya, termasuk *fixed mindset* dalam matematika. Hasil wawancara dengan guru sekolah menengah pertama di Indonesia juga menunjukkan bahwa siswa memilki masalah dalam operasi bilangan dan motivasi belajar matematika. Desain yang digunakan pada studi ini menggunakan studi kualitatif deskriptif berdasarkan hasil studi literatur, wawancara dengan guru mata pelajaran, dan pengalaman penulis sebagai pendidik. Studi ini juga menggunakan teori *zone proximal development* dan *mathematical mindset* sebagai bingkai dalam menyusun alternatif materi persamaan dan fungsi linear berdasarkan representasi visual dalam matematika. Hasil studi ini berkontribusi untuk memberikan alternatif materi untuk mengurangi hambatan tersebut menggunakan pendekatan representasi visual dalam matematika. Penulis juga memberikan alternatif seni mengajar untuk mengantisipasi hambatan yang muncul pada saat implementasi materi matematika berdasarkan representasi visual.

Kata kunci: Persamaan Linear, Fungsi Linear, Representasi Visual, Matematika

Abstract

Linear equations and linear functions are important materials for students in developing their in-depth knowledge of arithmetic and algebra. However, there is a gap between potential development and actual development, which causes obstacles for students in understanding the concepts of these two materials. This gap results in other obstacles, including a fixed mindset in mathematics. The results of interviews with secondary school teachers in Indonesia also show that students have problems in number operations and motivation to learn mathematics. The design used in this study uses a descriptive qualitative study based on the results of literature studies, interviews with subject teachers, and the author's experience as an educator. This study also uses the theory of zone proximal development and mathematical mindset as a frame for preparing alternative material for equations and linear functions based on visual representations in mathematics. The results of this study contribute to providing alternative materials to reduce these barriers using a visual representation approach in mathematics. The author also provides alternative teaching arts to anticipate obstacles that arise when implementing mathematics material based on visual representation.

Keywords: Linear Equation, Linear Function, Mathematics, Visual Representation.

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INTRODUCTION

Linear equations are a significant part of teaching algebra elements in secondary schools and play a role in students learning advanced mathematics topics (Wati & Fitriana, 2018). Algebra enables the representation and transformation of abstractions and produces mathematical tools that are powerful and useful in other disciplines, such as physics and chemistry (Mengistie, 2020; Holmludn, 2024). Students study algebra, starting with arithmetic operations and continuing to solve linear equations (Mengistie, 2020). In linear equation material, it may be the first time students experience the image of "=" after arithmetic, which shows the result of an operation and can handle all symbolic expressions (B Pirie & Martin, 1997). Even though it is considered an essential part of mathematics education, students from many countries show relatively negative attitudes towards algebra concepts (Mengistie, 2020). Computational problems and number knowledge significantly affect students' difficulties learning algebra concepts (Holmludn, 2024; Wati & Fitriana, 2018).

After linear equations, the following material is the concept of linear functions. The concept of linear functions is essential for starting analytical analysis and is the basis for further learning function graphs. The concept of linear functions is the first step in developing algebraic concepts (Pierce et al., 2010). In graphing linear functions, students learn about variables, parameters, each function in $y = mx + c$, the structure of function spaces and the properties of mappings between these spaces (Alt, 2016). In learning linear functions, material that requires slope analysis and graphs of linear functions is the most challenging material for students to learn (Postelnicu, 2011). Students often need help understanding the concepts of equations, graphs and gradients, so they cannot understand the relationship between these elements (Birgin, 2012).

After interviewing a teacher in Indonesia who teaches linear equations and linear functions, it was found that students face several obstacles when learning these concepts. One of the obstacles is related to understanding numbers. Additionally, students lack confidence in their math abilities, often unable to generate ideas and solve problems independently. According to the teacher, they struggle with comprehending algebra, linear equations, and linear functions as distinct concepts and fail to see the comprehensive connection between them.

The presence of obstacles to comprehension indicates the existence of a gap between the potential development and the actual development of students, also known as the zone of proximal development (Vygotsky, 1978). It means that middle school students should be able to understand the concept of numbers, but in reality, they still need to grasp it. Therefore, teachers must provide scaffolding or support to guide students through the zone of proximal development (Wood et al., 1976). Students' lack of self-confidence in algebra is driven by negative perceptions of the subject, leading to difficulties in understanding number concepts and fostering a fixed mindset due to the influence of learning (Maskar & Herman, 2024). Therefore, there is a need for learning with a mathematical mindset approach so that students have an optimistic view of mathematics learning (Boaler, 2022b). Apart from that, learning based on visual representation can be an alternative so that students can understand the concept of equations and linear functions and can connect it with other material (Boaler, 2022a). Context-based and visual learning is essential for mathematics education because mathematical concepts have their first roots in action, representation, and the physical-social world (Bruner et al., 1965). However, teachers' implementation of visual representation is often limited by their narrow understanding and needs improvement (Van Garderen et al., 2018). Therefore, students need to be allowed to practice external and internal representations in mathematical ideas and make representations a tool for thinking, explaining, and justifying that is driven by the norms created by the teacher (Pape & Tchoshanov, 2001). Visual representations of mathematics must be attached to a conceptual entity to be meaningful (Nardi, 2014). The study's results by Surya et al. (2013) also show that mathematical representation abilities can be improved using visual representation-based learning through a contextual approach.

Visual representation in mathematics is part of the method or way of presenting mathematical material that involves the teacher and the material in the didactic triangle called didactic-pedagogical anticipation (Suryadi, 2010). In internal didactic transposition (Lombard & Weiss, 2018; Bosch et al., 2021), didactic-pedagogical anticipation begins when the teacher prepares teaching materials and assignments to be given to students. The teaching materials are prepared based on textbooks derived from the existing mathematics curriculum. The transition process from textbooks to teaching materials is called knowledge to be taught (Chevallard, 1989).

Many obstacles occur in the transition process of learning, especially in mathematics. According to Prabowo et al. (2022), the obstacles that occur are caused by teachers who do not have in-depth knowledge of the school curriculum, methods, and materials to be taught. According to Nurlaily et al. (2019), teachers are usually reluctant to determine the problem context of the mathematics material to be taught, and the majority use problems found in textbooks. Apart from that, according to Cesaria & and Tatang (2019), students often experience ontogenically and epistemologically obstacles as a result of the questions and assignments given by the teacher.

Based on the information and analysis above, the research questions in this study include:

1. How can an alternative arrangement of materials on linear equations and functions be structured based on visual representation, zone of proximal development, and mathematical mindset?
2. What are the potential student responses to this alternative arrangement of materials on linear equations and functions?

METHOD

This study utilizes a qualitative design and a visual representation approach in mathematics to create alternative teaching materials for linear equations and functions. Qualitative research designs are useful for understanding complex phenomena, tracking unique and unexpected events, interpreting events, presenting different perspectives, and developing theories. (Sofaer, 1999). In addition, mathematics' distributed, visual, and physical nature has a positive impact. Most students who have experience learning mathematics abstractly need help interpreting and connecting mathematics learning. (Boaler, 2022b). This study also uses mathematical mindset theory to frame alternative learning materials. The mathematical mindset approach to learning significantly increases students' mathematics achievement and can change students' beliefs about themselves in relation to mathematics learning (Boaler et al., 2021).

Figure 1 shows the research flow used in this study.

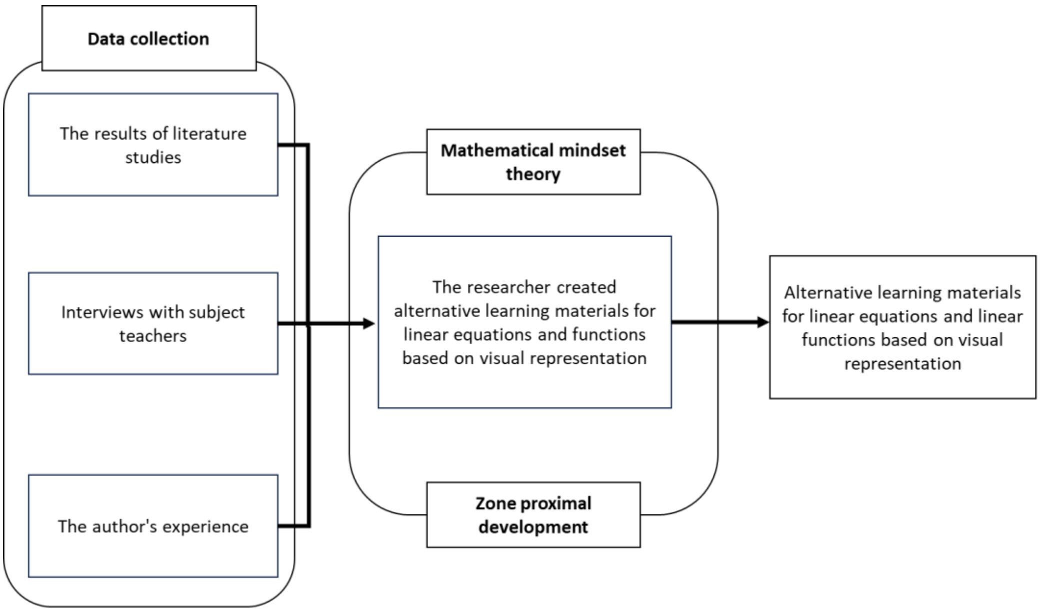

Figure 1. Research Flow

The author created alternative learning materials for linear equations and functions based on the results of literature studies, interviews with subject teachers, and the author's experience as an educator. The subject teachers interviewed came from one of the junior high schools in

Indonesia that taught equations and linear functions. The author also prepares anticipated obstacles from the prepared alternative linear equations and linear functions based on the information above.

The course material consists of assignments that students need to complete and present. This allows for discussion and the exchange of ideas among students. The study utilizes a visual representation approach based on key observations (Boaler, 2022a): The assignments are challenging but accessible to students. This means that the assignments must follow the student's abilities, at least following the student's development potential (Vygotsky & Cole, 1978). However, assignments must also be challenging, meaning not too easy, so as not to foster a fixed mindset (Boaler, 2022b). Students view the assignment as a puzzle. The assignment must invite students' curiosity about the solution so that students can try to complete it. The material in the assignment does not have to be in the form of a real-world context but must attract students' attention to foster open thinking and mathematical connections. Visualizing growth patterns can help students understand how patterns develop. Assigning tasks within the context of growth patterns can help students develop thoughts and ideas that lead to general expression. Most importantly, students can discover these concepts for themselves through patterns. Students are encouraged to realize that they have developed their way of looking at pattern growth to develop different method ideas while still providing valid solutions. Assignments should allow students to make mistakes, and teachers should encourage students to recognize and correct those mistakes.

RESULTS

Seeing Linear Equations

The Form $y = mx$

Figure 2 is a type of growth pattern with the aim of students being able to see linear equations in one variable. Students are asked to continue the pattern, going up and down like a puzzle. The student may easily continue the upward pattern and initially experience some difficulty when attempting the downward pattern. Students can guess that the 0th shape is that there is not a single square there. However, various opinions will arise when students are asked to show patterns of negative numbers. This difference of opinion is something good and will give rise to a thought process. Teachers only need to facilitate students and give instructions to students to respect all existing opinions.

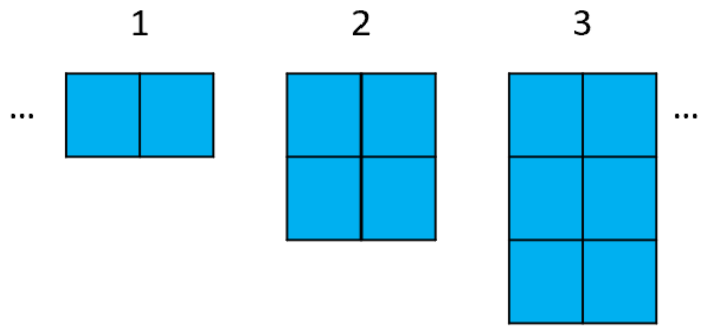

Figure 2. Growth Pattern in The Form $y = 2x$

Teachers can use Table 1 to help students summarize the general form of a growth pattern. Once students have completed Table 1, the teacher can ask them to predict the 80th pattern. Students may suggest different ways to reach the 80th pattern until the teacher guides them toward making general conclusions about the shape of the growth pattern, which in this case, is $y = 2x$. Initially, students may develop ideas to discuss in class before arriving at the formal form with the teacher's help.

Table 1. Number of Squares in The Growth Pattern of The Form $y = 2x$

-3	-2	-1	0	1	2	3	4	5	6
…	…	…	…	2	4	6	…	…	…

In the end, students need to be free to identify their own growth patterns, and they should do this in groups. Afterward, each group can share their respective growth patterns. Some students may choose simple patterns like $3x$, $4x$ and $5x$, while others might be interested in patterns with negative coefficients, such as $-2x$. If no students explore patterns with negative coefficients, the teacher can pose a question to make the presentation discussion more challenging.

The Form $\boldsymbol{y = mx + c}$

Similar to the growing pattern of the $y = mx$ shape, the idea of the $y = mx + c$ shape is to add one square that never grows up or down. In Figure 3, the rectangle is orange. The orange rectangle represents the constant value in a one-variable linear equation. Similarly, students are asked to continue the pattern, up and down and create a general shape from the pattern. The general form that should be obtained is $y = 3x + 1$.

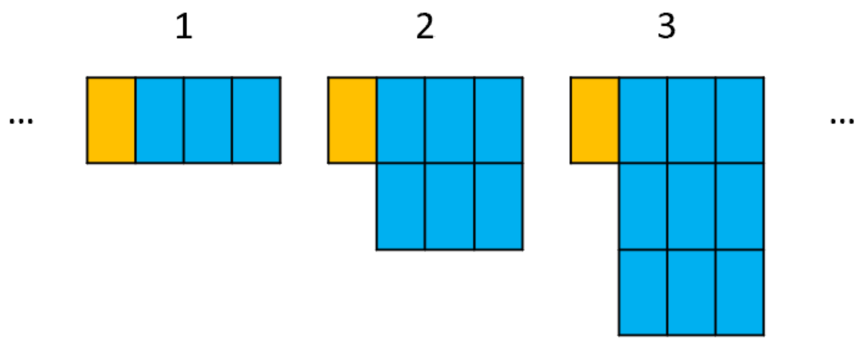

Figure 3. Growth Pattern in The Form $y = 3x + 1$

Students are also encouraged to create similar patterns to deepen their understanding and engagement in learning and to continue with presentations. Teachers should ask challenging questions to raise the level of difficulty when students present, as discussed above. In this section, teachers can also use discussion sessions to explain growth patterns in the negative number domain, helping students understand the distinction between positive and negative growth patterns and the relationship between the two.

Seeing Linear Function

The Form $y = mx$

The linear equation material discussed above leads to a study of linear functions. In Figure 4, students have grasped the growth pattern represented by the equation $y = 2x$ for values ranging from -3 to 4. Using this pattern, the teacher can pose challenging questions related to the various patterns that can be created. Students can be prompted to continue the pattern, either upward or downward, as they prefer. Subsequently, students are asked to present their findings, and the teacher can elucidate the relationship between the student's work and the domain of the linear function. The teacher can also pose challenging questions such as, "Is there a pattern between the values 1 and 2, or between -2 and -3?" This question serves as a prompt for the teacher to explain the domain material in real numbers, highlighting that the domain of a linear function is not always restricted to integers.

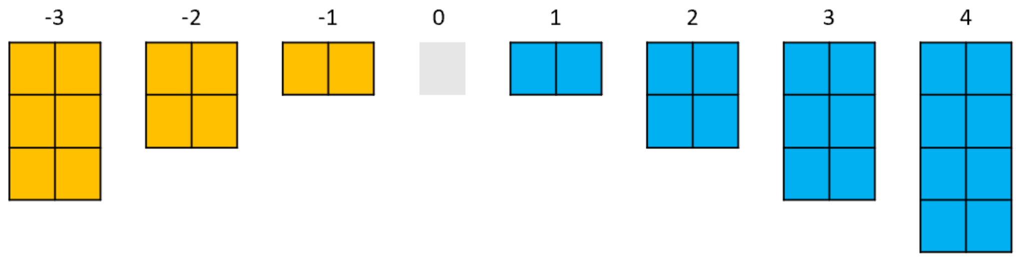

Figure 4. Growth Pattern in The Form $y = 2x$ Domain -3 To 4

After distributing shapes to students, they were instructed to arrange them on a checkerboard diagram, later revealed to be a Cartesian diagram. This process served as a graphical representation of a linear function (see Figure 5). Initially, students proposed various graphical ideas based on the shape placements from Figure 3. However, the teacher ultimately clarified the concept by presenting a standard drawing, resulting in a clear graph of a linear function, as depicted in Figure 5. Removing the blue and yellow square shapes allowed students to see the graph of a standard linear function.

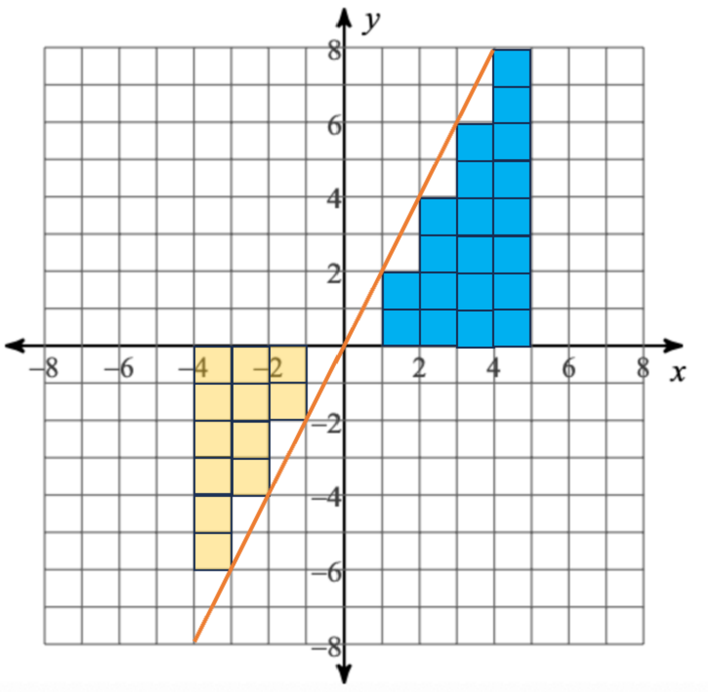

Figure 5. Graphical Representation of Linear Functions in The Form $y = 2x$

In Figures 4 and 5, the teacher can ask questions about the amount of change that has occurred. After getting input from the students on the pattern of change, the teacher can clarify and observe the pattern in the graphical representation of a linear function as the slope of a line, also known as a gradient. Teachers can also have students create a graphical representation of a linear function for each answer to the equation form that was previously created to help them better understand the relationship between the rate of change and the slope of a line.

The Form $y = mx + c$

After students have grasped the structure $y = ax$, they can examine the domain, rate of change, and graph of a linear function of the form $y = ax + b$. In Figure 6, the growth pattern of the equation $y = 3x + 1$ is presented with a domain ranging from -2 to 3. Similarly, students are encouraged to analyze this equation in various other domains, using both integers and real numbers. That way will help enhance students' comprehension of linear function domains.

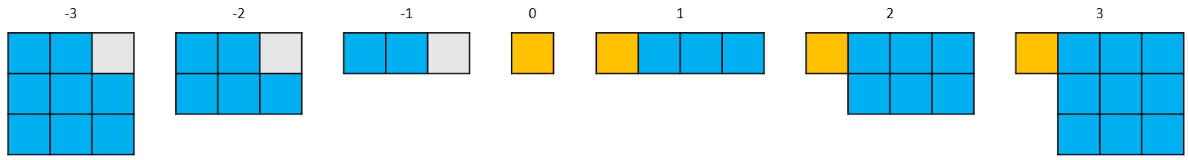

Figure 6. Growth Pattern in The Form $y = 3x + 1$ Domain -3 To 3

Figure 7 shows a graphical representation of a function of the form $y = 3x + 1$. Previously, the teacher could give students freedom in advance in determining the form of the linear function graphic representation of the previous growth pattern. Students may have difficulty placing squares in a growth pattern on a Cartesian diagram and producing the correct graphic shape. However, teachers can use discussion sessions to guide students to the appropriate form of representation and provide logical reasons for this form of representation.

Through the process above, the teacher can explain the domain and slope of the line again. However, in this section, the teacher can also explain the meaning of constant values in a linear function. In simple terms, Figure 7 shows that the constant value provides additional value to the codomain analytically. This value shifts the graph of the linear function

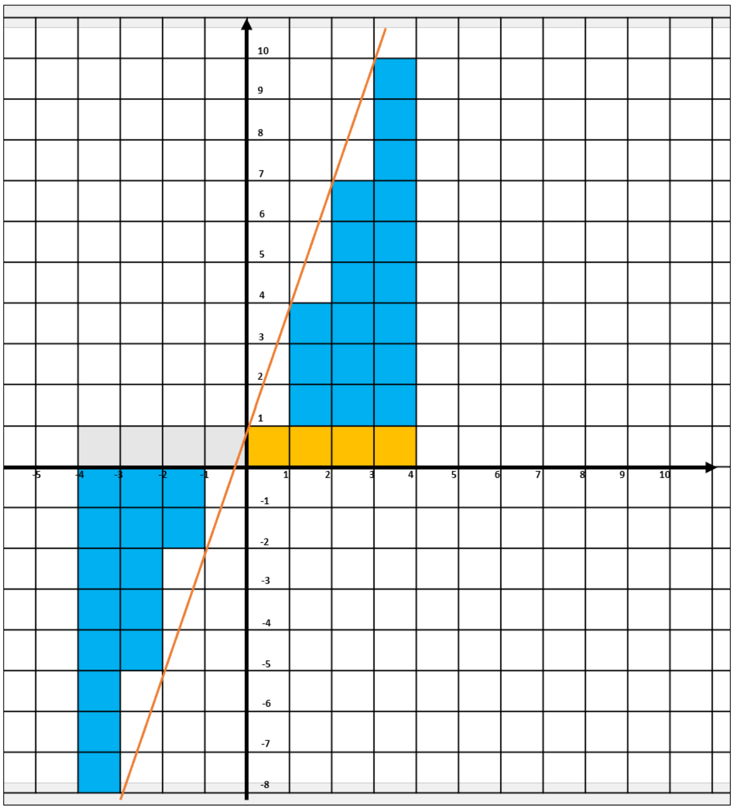

Figure 7. Graphical Representation of Linear Functions in The Form $y = 3x + 1$

DISCUSSION

The following section considers the potential implications of the alternative material on linear equations and linear functions described earlier. These implications are based on a study that integrates relevant theories, teacher interview findings, and the author's experience as an educator.

During the process of implementing learning using growth pattern representations to explain linear equation material, students may first experience difficulties. Maskar and Herman (2024) showed in their study that students are more accustomed to procedural mathematics, so they experience learning difficulties when the process is changed into a representation that requires them to think about their mathematical ideas. Therefore, Boaler (2022b) provides several important observations so that students can become familiar with mathematical representations. First, the task given must be challenging but still accessible to students, and students can see it as a puzzle game. Next, students should be allowed to make mistakes and should respect every idea and opinion that they provide. Finally, create heterogeneous groups and avoid selecting groups with students who have the same abilities so that students do not have a fixed mindset (Boaler et al., 2021) on the material.

Furthermore, some students may experience difficulties because of their learning history. This learning history causes problems in students' potential development (Vygotsky & Cole, 1978), resulting in a gap in students' potential development in learning mathematics, resulting in difficulties in developing their actual abilities (Christmas et al., 2013). Based on the results of an interview with one of the teachers who taught linear equations and linear functions, the teacher explained several obstacles students had in learning this material. The main obstacle is students' difficulty in number operations, especially negative numbers and fractions. Students' difficulties in carrying out number operations affect the way students understand linear equation material further. In a further interview, the teacher also revealed that these difficulties meant that the teacher had to explain the basic material back to the students before going into the core material, thus causing a longer learning time and having a domino effect on subsequent material.

Referring to Vygotsky (Shabani et al., 2010), what these students experienced was an obstacle to potential development, where mentally learning students should have been able to reach the material. However, they apparently did not understand the material. Therefore, there is a gap between potential development and actual development (Podolskiy, 2012). Therefore, to fill this space, there needs to be an appropriate learning design based on the obstacles experienced by students and other learning obstacles. Representation-based learning that allows students to reach abstractions in mathematics learning is an alternative solution to fill this space so that students are more proficient in mathematics (Mainali, 2021; Pape & Tchoshanoy, 2001). Preparing increasing patterns, both upward and downward, is indirectly a learning process that can fill students' development potential, and also, students can understand linear equation material. The downward growth pattern for negative numbers is also an alternative used so that students can understand integers and their operations, which may be a type of learning that they did not get before. The result of this learning history is that students are unable to understand and connect this material to other mathematical material. Mathematics learning can take root in students when students feel that the learning has meaning for them (Henningsen & Stein, 1997; Boaler et al., 2022).

Figure 6 shows a growth pattern where students have to arrange patterns with additional constant shapes that must always be present in each pattern and accompanied by the growth of other shapes. Challenges occur when students have to continue the pattern of negative numbers. This is because the pattern will be very different from the positive shape in Figure 4. The correct pattern is in Figure 6. The majority of students may form the same pattern as Figure 4, but this will be problematic because it will not match the results of the number operations. In addition, this pattern also does not show the role of constants. At this moment, the role of teachers is very necessary. The teacher's role in closing the gap between actual development and potential development (proximal zone) is providing modeling, feedback, questions, teaching, and cognitive structuring (Sanders & Welk, 2005). Therefore, teachers need to guide students toward the right pattern. However, in the process, the teacher must not go too far in telling the actual pattern. In theory, this is called a didactic contract, which is an important element in a teaching and learning situation that can support or create obstacles in the acquisition of new knowledge by students (Carmo et al., 2020). As a result, students only need to be asked questions that direct them to the right form so as to avoid creating new learning obstacles if students are immediately given solutions. One method that can be used in asking questions is the Socratic Method (Rhee, 2007; Sorvatzioti, 2012).

It is important to recognize that once this information is presented to students and students engage in a series of learning and discussion activities, the teacher must explain the material based on scholarly knowledge, or at least, according to the textbook being used. Ultimately, learning mathematics requires explanations that follow correct procedures, but they must be given at the right time (Schwartz & Bransford, 1998; Findell et al., 2001). Nevertheless, with the initial stimulus, it is hoped that students can develop a sense of learning in order to engage in reasoning in the subsequent stages of learning.

The representations of linear functions in Figures 5 and 7 build upon the concepts covered in the linear equation material. Once students grasp the concepts of linear equations, they move on to new material, specifically linear functions. By continuing with graphing linear functions, the teacher expects students to comprehend the relationship between linear equations and the graphs of linear functions and to identify the differences between the two. Representations of linear functions also provide students with their initial exposure to analyzing the form of linear functions, starting from understanding differences or slopes, variables, and constant values.

The next obstacle, based on the results of interviews with teachers, is related to student motivation. The teacher may have designed good material and planned a pedagogical process that suits the students. However, it ends up not being optimal or even not going according to plan due to students needing to be more active and showing interest in learning at all. There are several reasons why this could happen. However, what happens most often is that students' negative perceptions of mathematics learning give rise to a fixed mindset towards students. Having a fixed mindset means believing that intelligence is fixed, and this can contribute to a decline in students' academic performance, including interest and participation in learning mathematics (McNabb, 2021; Jaffe, 2020).

The mathematical representation above regarding equations and linear functions is an alternative that can be used to foster students' growth mindset. When students provide their ideas and feel directly involved in learning mathematics, it can foster a growth mindset. Apart from that, students' involvement in mathematics and their feeling capable of doing it will foster mathematics identity and increase motivation to learn mathematics. Student Growth Mindset orientation can predict student engagement and achievement (Bostwick et al., 2017; Setiawan et al., 2021). However, a growth mindset does not directly influence student achievement in learning mathematics; the influence of a growth mindset is influenced by other predictors, including intrinsic motivation, mathematical identity, self-efficacy, and mathematics anxiety (Dong et al., 2023; Boaler, 2022b).

CONCLUSION

Alternative material on linear equations and linear functions based on visual representation in mathematics, one of which is using growth patterns. Growth patterns can be used as a challenging task for students but still within the range of students' abilities. Through growth patterns, students can connect their understanding of numbers and algebra to linear equation material and also connect linear equation material to linear functions. Visual representations also help students be directly involved in learning through students' ideas in continuing growth patterns to find generalizations of a linear equation and graphs of linear functions.

Student involvement in learning can grow students' self-confidence, bring out students' mathematical identities, and also make students view mathematics as a meaningful science. In the end, it will foster students' mathematical mindset. Basically, junior high school students can understand linear equations and linear functions. However, many students still need help with the previous material and even number operations material. Therefore, teachers need to combine this alternative material with the art of good teaching. The author suggests that teachers can combine it with mathematical mindset theory.

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